Franklin W. Olin College of Engineering

# A Faraday Cage Exploration:

## Final Report and Project Assessment

Project Developers: Andrea Lai, Sandra Lam, and Rachita Navara

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### *Introduction: Why Faraday cages?*

In this project, we developed a hands-on activity and related demonstrations that focus on the shielding property of Faraday cages, which are metal enclosures that can block out electromagnetic waves. Faraday cages are valuable tools for explaining electricity and magnetism to students because they produce intriguing, easily observable phenomena while applying an understanding of various physics concepts. Students can test electromagnetic shielding in a fun and simple manner by placing a communication device inside a conducting container, closing the lid, and observing whether the device is immediately silenced. The ease of making and testing Faraday cages with easily obtainable materials is conducive to the classroom atmosphere for which this project was designed. The cage system can also accommodate further complexity, given that varying conducting wall thickness and continuity will affect shielding performance.

Moreover, the Faraday shielding effect often emerges in real-world scenarios. An understanding of these cages can elucidate several fascinating concepts, from why cell phone reception is so weak in elevators, to how high-security organizations use metal rooms to prevent electronic espionage [1]. The various manifestations of electromagnetic shielding offer students opportunities to explain natural phenomena with their own scientific knowledge. This prompts students to adopt a self-motivated, inquisitive mindset that can extend through their educational career and beyond.

### *Project Goals and Objectives*

An underlying goal of our project was to further our own understanding of electricity, conductor behavior, and electromagnetic waves. To attain a deeper comprehension of these topics, we qualitatively and quantitatively analyzed Faraday cage behavior and developed explanations for cage performance in response to different electromagnetic signal frequencies. We were then able to compose a scientific exploration for middle school students that reveals and justifies these differences in Faraday cage effectiveness. The step-by-step testing and "learning by doing" approaches in our hands-on activity encourage students to explain for themselves why different cages produce different results.

According to Massachusetts curriculum standards, students are not exposed to the topic of electromagnetic waves until high school introductory physics [2]. However, we sought to introduce the concepts of electricity and magnetism to students before high school. Because the abstract concepts of electromagnetism may be intimidating to students when first encountered,

our activity aims to provide familiarity with the topics in an interactive setting before students can develop a mentality that "physics is hard" [3]. For instance, we preface our explanation of electromagnetic waves by demonstrating wave propagation in a mechanical model such as a Slinky. In this way, students are also engaged to learn advanced science topics within the context of real-world applications.

### *Specific Pedagogical Approaches*

The broader impact of our work lies in the pedagogical approaches we incorporated in our hands-on activity. By encouraging middle-school students to rationalize complex physics phenomena through an exciting, applicable investigation of Faraday cages, we can instill a self-motivation to explore science creatively.

While developing this activity, we consulted several pedagogical resources that mirror our project intentions. In particular, the publication, "Learning Electromagnetism with Visualizations and Active Learning" addresses the specific physics content we will explore with our demonstration [4]. The paper focuses on learning electromagnetism with an emphasis on "concrete and visual representations in teaching abstract concepts." Accordingly, our mechanical wave demonstrations and simple analogies are aimed to give students the clearest explanations of abstract topics, about which they may have minimal background knowledge. These analogies will be especially valuable for students performing this activity, given that similar instructive models have been proven successful in a study that tested the effectiveness of teaching electromagnetism by analogy [5].

### *Content to Follow*

To further our own learning and student understanding, we have prepared background materials that are appropriate for our middle school audience. This set of readings includes student-friendly introductions to electricity and wave motion. Prior to the in-class activity, teachers may assign one or both of these materials for students to read, as they see fit. These student introductory reading materials begin on the next page of this report, followed by an instructor copy of our hands-on activity procedure. Appendix A of this document contains an activity sheet hand-out for students. Next, Appendix B includes a companion lesson to our activity, which illustrates wave motion as applicable to Faraday cage electromagnetic shielding. Appendix B includes a wave lesson that supplements our activity. Finally, the advanced physics content we explored while developing this activity is included in Appendix C.

## *Introductory Reading, part I*

*Electricity and Conductors*

Have you ever experienced an electric shock after walking across a carpet and touching a doorknob?
Has anyone told you that the safest place during a lightning storm is inside your car?
Do you know why cell phone reception is so weak inside an elevator?

All of these puzzling concepts can be explained by electricity, and its behavior within metals. Let's begin with the first topic: the painful shocks that result from **static electricity**, a build-up of electric charge. The atoms that make up some materials hold onto their electrons strongly than those in other materials. When dissimilar materials such as wool and plastic are rubbed against each other, electrons are pulled from one material and taken up by the other [1]. As a result, one material becomes more negatively charged because it gains more electrons, while the other becomes more positively charged because it loses electrons. Consider the carpet-sock example: when you drag your feet along the carpet, you pick up extra electrons from the floor, giving you a negative charge. Then, when you touch a metal doorknob, the electrons jump from your hand to the doorknob so that both objects can share the same charge. These free electrons will travel through the air in an electron "river," forming a visible spark [2].

The speed with which your electric charge is transferred to the doorknob demonstrates the properties of a **conductor**, any material that promotes the flow of electricity. Poor conductors (or *insulators*) restrict the flow of electricity. Metals are especially good conductors because atoms in a metal do not hold tightly to their electrons [3]. Because these electrons can move freely within the metal, they can transfer, or *conduct* electricity very quickly.

This leads us to our second question: why would you be protected from lightning when sitting inside a car? Might it be because the car has rubber tires, and rubber *insulates* objects from electric charge? In truth, the rubber offers no protection from a lightning strike, which is simply a high-energy electric spark much like the one you feel when touching a doorknob. When you think about it, lightning can travel through the air—a relatively good insulator. If air cannot protect people from lightning, a few inches of rubber certainly can't shelter the occupants of a car from a lightning bolt. So what's the real reason that cars are safe during a lightning storm? The answer lies in the behavior of conductors in response to electric fields.

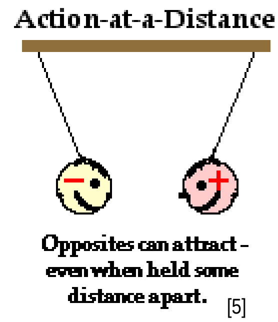
*Electric Fields*

Let's consider a rotten egg placed in the corner of a room. You can imagine how smelly the room would become, and your nose would soon detect the presence of the egg. Essentially, the egg has produced a "stinky field" within the area. As you move toward the rotten egg, you are affected by the stinky field more and more strongly. We can think of an electric charge like a rotten egg because every charge produces an **electric field** that affects other charged objects within it (similar to the way noses are affected by a stinky field) [5,6].

In our stinky field analogy, we wouldn't willingly approach the rotten egg, but electric charges in fields don't always move away from the source of the field. Have you ever heard the phrase,

“Opposites attract; likes repel”? This applies to electric charges! If both objects share the same sign charge, the objects repel. If both objects are oppositely charged, they attract each other, so the charged object will move toward the source of the electric field. Note that charges don’t have to come in contact to affect each other; the attractive or repulsive force between charges acts across the distance between them. This is why electric force is often termed an “action-at-a-distance force,” exerted by an electric field on other electrically-charged objects [7].

It may be difficult to understand this concept, but other familiar forces act at a distance. An excellent example is Earth’s gravitational field. What happens when you hold an apple in the air and let go of it? Just as Isaac Newton observed centuries ago, it falls to the ground. Of course, the attractive force that the earth exerts on the apple is invisible—just like attractive and repulsive electric forces. Action-at-a-distance forces also apply to magnets. If you have ever played with two magnets, you know that holding them apart in a certain orientation causes them to snap together, while they refuse to come together when you flip one magnet over. Just like with electrically charged objects, opposite magnetic poles attract, and like poles repel. Further, the magnets only respond to each other when they are close together, revealing the presence of magnetic fields.



[5]

*Faraday Cage Shielding*

Now that we understand electric fields, we can fully explain why cars can protect you from lightning. Recall that electrons can move very quickly inside a conductor, and that lightning is made up of many electrons. So let’s examine how a conductor behaves in response to an external electric field. Consider a conductor that is negatively charged, or contains excess electrons (-). These “extra” electrons repel each other because they are of the same sign. Thus, they try to get as far away from each other as they can. In order to do this, they arrange themselves on the outside surface of the conductor, which ultimately causes the electric field inside the conductor to be cancelled [8]. Imagine ten of your classmates stuffed inside an elevator after soccer practice. Everyone would smell sweaty and try to move as far apart as possible. This would be best achieved if everyone pressed themselves against elevator walls—just as electrons redistribute themselves along the outside surface of a conductor.

So what happens when lightning strikes a metal car? Due to the electric field created by the lightning strike, electrons within the metal car body will redistribute along the outer surface of the car. It turns out that this electron redistribution cancels the electric field inside the car. This process is known as **electrostatic shielding** (electro*static* = the electric charges aren’t moving). Thus, because Faraday cages contain regions of zero electric field, people are protected from a lightning storm within metal enclosures, such as cars and steel-framed buildings [9].

Now that you understand that Faraday cages have no internal electric fields, you can move on to Part II of this reading to discover how cages can block waves such as radio signals.

**Part I Introductory Reading References:**


1. Pilkington, G.A. “Static electricity-friend or foe.” Electrical Safety in Hazardous Environments, 1994. Fifth International Conference on Electrical Safety in Hazardous Environments, 19-21 (1994): 63- 68.
2. “Static Electricity & Static Control.” 2008. ElectroStatics, Inc. 18 Apr. 2008 <http://www.electrostatics.com/page2.html>.
3. Voltmer, D.R., "Transient response of good conductors." Proceedings of the IEEE, 63, 2 (1975): 318-319.
4. Murphy, Tom. “Faraday Cages and Microwaves.” UCSD: Physics 8; 2006. 16 Apr. 2008 <http://physics.ucsd.edu/ ~tmurphy/phys8/lectures/14_microwave.ppt>.
5. Henderson, Tom. “Lesson 4: Electric Fields.” The Physics Classroom Tutorial. 2007. 18 Apr. 2008 <http://www.glenbrook.k12.il.us/GBSSCI/PHYS/CLASS/estatics/u8l4a.html>.
6. “Electric field.” Encyclopaedia Britannica. Electronic ed., version 1.0. 2005.
7. Crowell, Benjamin. “10.1.3: The Electric Field.” Simple Nature. Vol. 2. Creative Commons, 2007. 483-490.
8. Duffy, Andrew. “Electric Field.” Boston University: PY106 Elementary Physics II course notes. 2000. 18 Apr. 2008 < http://physics.bu.edu/~duffy/PY106/Electricfield.html>.
9. Centre for Earth Science Studies. Lightning: Phenomenon & Precautions. Thiruvan Anthapuram, 2002.
10. Kaiser, Kenneth. Electromagnetic Compatibility Handbook. Boca Raton: CRC Press, 2005. 24-75.
11. Crowell, Benjamin. “Chapter 11: Electromagnetism.” Simple Nature. Vol. 2. Creative Commons, 2007. 571-578.
12. Malyapa, R.S. et al. 1997. “Measurement of DNA damage after exposure to electromagnetic radiation in the cellular phone communication frequency band (835.62 and 847.74 MHz).” Radiat. Res. 148, pp. 618–627.

## *Introductory Reading, part II*

*Physical Waves*

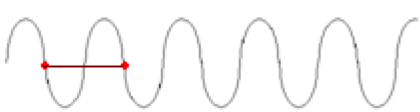
From Part I of this reading, we discovered how a Faraday cage can cancel the electric field inside it, but how does a Faraday cage behave in the presence of waves? To begin, let us look at the different properties of visible waves.

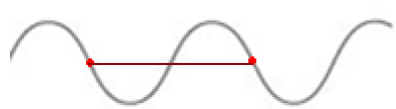
Have you ever observed the motion of ocean waves? What about ripples that form when you drop a stone in a lake? At first, waves only appear where the stone was dropped. However, as time passes, these waves circle further and further out from their center.

http://it.rit.edu/~ero/psWorkshop/ripples.jpg

This reveals that waves can move from one point to another. Can waves also carry something with them? Let's think back to our pond example; suppose a leaf is floating on the surface of the water. As the ripples pass by the leaf, the leaf only bobs up and down. Because the object moves in the vertical direction as the wave passes in the horizontal direction, the result is a **transverse wave**.

Keep in mind that not all waves are equally stretched out. For example, look below at the images of waves. How would you describe the difference? Both waves have the same height, but their **wavelengths—**the distance between two peaks or two dips—are different. In the picture below, the horizontal lines span the wavelength for each wave. You can see that the wave pattern repeats more often in the left picture than the right one.

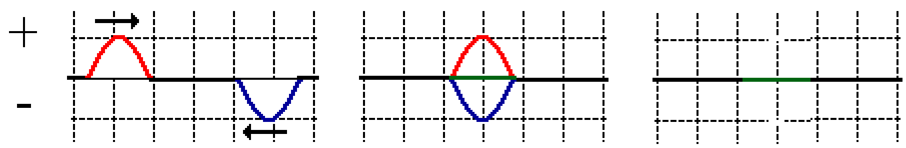




Suppose that both waves are moving to the right at the same speed. In this case, it takes the same amount of time for the two waveforms above to pass a fixed point. Let's assume that it takes both waveforms 10 seconds to pass by this point. Then, looking only at the waveform on the left, we can say that 5 waves pass in 10 seconds, or 5/10 waves per second. This number describes the wave's **frequency**, a measure of how many times a wave pass by a point over some length of time. This is easy to remember because the word "frequent" describes *how often* some activity happens. We can also describe a wave's motion by its **period**, which is the time it takes for one

full wavelength to pass through a point. Mathematically, the period is the inverse of frequency. Therefore, in 10 seconds, 5 waves pass by a point. This means that it takes 10/5, or 2 seconds for one wave to pass the fixed point. Now, let's take a look at the waveform on the right. How do its frequency and period differ? By looking at the shape of the entire wave, you can probably tell that fewer waves fit in the same length of the other wave. This means that the wave has a lower frequency and greater period because it takes longer for the same amount of waves to pass by a fixed point.

One other aspect of waves to consider is how two waves can affect each other. Let's consider two waves that travel in opposite directions:




When two waves meet, the resulting wave is a combination of both of them. The specific waves shown above are of the same size but opposite sign, so they cancel each other out. You can think of this as the result of adding 1 and -1.

*Electromagnetic Waves*

Not all waves are visible like the ones we explored in the previous section. How do you think our cell phones and radios receive signals? These signals are actually in the form of invisible waves! Although the physical waves we described require some substance (such as water) to travel through, **electromagnetic waves** can travel through empty space. What do you think produces these waves? First, look at the word *electromagnetic*—a combination of the words electric and magnetic. This suggests that electromagnetic waves involve electric and magnetic fields.

So do these two fields interact? Remember that an electric field is defined as the region around an electric charge. As electrons speed up or slow down, the electric field they create will also change. This changing electric field actually creates local changes in the magnetic field that originates from the Earth's magnetic poles. Interestingly, this changing magnetic field also changes the electric field. Ultimately, the electric and magnetic fields continually alter each another, forming an electromagnetic wave. You can picture this behavior by considering a perpetual motion machine (below). If you pick up and release a ball on one end of the row, after a few seconds, the ball on the opposite end will swing out. Eventually, the first ball moves again. This continual back and forth motion between the first and last ball of the perpetual motion machine is similar to the relationship between the changing magnetic and electric fields: a change in one causes a change in the other.

These electromagnetic waves actually exist all around us. For example, radio and cell phone signals travel through the air as electromagnetic waves. In fact, radios pick up waves of different frequencies, which correspond to different stations. When you change the dial of the radio, you are actually deciding which radio waves you want your radio to translate to sound!

*Electromagnetic Shielding*

So why do you think a cell phone would have weak reception in an elevator? Let’s examine the structure of an elevator. Like a metal car that protects its occupants from lightning, the elevator is also a closed metal container—a Faraday cage. If the cell phone signal is low inside an elevator, this suggests that the signal waves to the phone are blocked. How might this happen? As the electromagnetic wave approaches the Faraday cage, the wave excites the electrons on the surface of the conductor, causing them to move rapidly. Now recall what accelerating charges produce—an electromagnetic wave! This generated wave, traveling away from the metal cage, has an interesting relationship with the incoming cell phone signal. Like we saw earlier, two waves traveling in opposite directions can cancel each other out if they have same size, but flipped orientation. We know that signal-carrying electromagnetic waves cannot reach a cell phone inside an elevator, so what must be happening? The wave generated by the moving electrons on the elevator surface actually cancels out the incoming cell phone signal.

What do you think would happen if the walls of a Faraday cage had holes? Would incoming waves be able to sneak in through the holes? As long as the electrons on the surface can move around the holes, they can produce an electromagnetic wave that will cancel the incoming wave. However, holes do slow down electron flow—much like how rocks in a stream slow down water flow. Will this affect the performance of the cage? Well, if the incoming wave has a high enough frequency, the electrons on the conductor cannot redistribute themselves fast enough to block the incoming signal [3, 4].

So what might improve the performance of a Faraday cage? As we just explored, the more continuous the conductor walls are, the better they can cancel incoming waves. Thicker walls also improve the cage’s ability to block waves, because they can hold more electrons. In turn, more electrons can move around to cancel the incoming signal. We can compare the electrons to soldiers defending a castle – if more soldiers (electrons) are present, the better they can keep outside invaders from coming inside. Alternatively, a building with more stories (or a thicker conductor wall) can hold more people, or more electrons. An interesting property of this phenomenon is that it takes a greater conductor thickness to cancel low frequency waves. For this reason, thicker cage walls are needed to block AM radio signals, which have a lower frequency than FM signals.

The performance of the Faraday cage is affected not only by properties of its conducting surface, but also by the effectiveness of the radio receiver and the strength of the original transmitted signal [5]. Keep in mind that touching the radio antenna to the inner wall of the mesh cage basically connects the radio to the electromagnetic field that the cage is trying to keep out since both the antenna and the metal cage are conductors. Thus, touching your antenna to the cage wall gives the electromagnetic wave a direct path to the radio and decrease the degree that the signal is blocked.

Where else might we see a Faraday cage at work? Elevators, for example, are essentially large metal cages – within them, cell phone signals are less able to reach us, and our phones frequently lose service within elevators for this reason. Not all effects of these cages are nuisances, though! Microwaves work on a similar principle, but instead of keeping waves out, they keep electromagnetic waves trapped *within* the enclosure. Faraday cages are also used to prevent electromagnetic interference for certain medical test procedures. Patients are protected from harmful radiation when inside a metal enclosure, so this set-up is often used in places such as hospitals and research labs [6]. Take a look around sometime and see where you can find a Faraday cage in action!

**Part II Introductory Reading References:**


1. Henderson, Tom. “What is a Wave?” The Physics Classroom Tutorial. <http://www.glenbrook.k12.il.us/gbssci/phys/Class/waves/wavestoc.html>
2. “A Science Odyssey: Radio Transmission” PBS Online, 1998 <http://www.pbs.org/wgbh/aso/ tryit/radio/radiowaves.html>.
3. Murphy, Tom. “Faraday Cages and Microwaves.” UCSD: Physics 8; 2006 <http://physics.ucsd.edu/ ~tmurphy/phys8/lectures/14_microwave.ppt>.
4. Rojansky, Vladimir. Electromagnetic Fields and Waves. New York: Dover Publications, 1979.
5. Kaiser, Kenneth. Electromagnetic Compatibility Handbook. Boca Raton: CRC Press, 2005.

# A Faraday Cage Exploration: Instructor copy

*In an exploration of electric fields, conductors, and real-world manifestations of Faraday cage shielding, students construct a metal mesh enclosure to demonstrate that a closed conductor can block electromagnetic waves. Students discover that based on the cage wall thickness and continuity, the cages can block radio signals of different frequencies.*

## GRADE LEVELS

*This scientific exploration serves an initial exposure to the concepts of electricity and electromagnetic waves, appropriate for grades 6-8; INCLUDE*

## SCIENCE TOPICS

*-electricity and electric fields*
*-conductors*
*-electromagnetic shielding (Faraday cages)*

## SKILLS DEVELOPED

*-applying knowledge of science topics to make design choices*

*-relating concepts from a visual mechanical model to the abstract concepts of electricity*

## TIME REQUIRED

| | | |
|---|---|---|
| ***Setup*** | *if pre-building cages* | *1-2 hrs* |
| | *if allowing students to build cages* | *20 min* |
| ***Performance*** | *Introduction to material for students without background in science topics* | *1-2 hrs* |
| | *Introduction to material for students w/background knowledge* | *30 min* |
| ***Clean-up*** | | *10 min* |

***Pedagogical Value of this Exploration:***

| | *Short-term Objectives* | *Long-term Goals* |
|---|---|---|
| For Students: | • Understand (in simple terms) phenomena from static electricity to electromagnetic shielding, and be able to justify why these concepts occur in nature, from the basic level of electron behavior<br>• Be able to incorporate group opinions into the design of one cage, per group<br>• Apply concepts explained in the Introductory reading to design an effective cage. They should be able to predict why certain signals will or will not penetrate the enclosure | • Learn how to integrate various physics concepts into the design and assessment of a testable application (Faraday cage)<br>• Learn how to work collaboratively on a single group science project<br>• *Educational impact:* develop an interest in why natural phenomena occur, and pursue an understanding of these concepts by asking questions and conducting self-driven investigation |
| For Teachers: | • Help further student understanding of electricity/magnetism physics by explaining content with student-friendly analogies<br>• Further own understanding of the specific concepts applicable to this activity (how different waves penetrate a metal enclosure, depending on amplitude and frequency, etc.), by referencing Appendix C, which includes advanced physics content for the instructor<br>• successfully address potential student misconceptions about Faraday cage applications, including why cell phones do not receive a signal in elevators, why a car acts as a partial Faraday cage during a lightning storm, and why only certain radio signals can penetrate a metal enclosure | • Gain practice in translating technical, higher-level physics concepts into non-technical, simplified terms<br>• Help demystify abstract physics concepts, in order to make physics more accessible for students<br>• *Educational impact:* harness students' curiosity about real-world phenomena to motivate them to explore physics topics |

## *Materials Needed:*

- ☐ Metal Mesh (a variety of different metals/mesh sizes would be valuable)
- ☐ Cardboard Box
- ☐ Radio with both AM and FM capabilities
- ☐ Cell phone
- ☐ Baby monitor
- ☐ Packing tape and/or hot glue gun
- ☐ Scissors

## *Safety*

The metal mesh can be dangerous for students to use because of the sharp edges, which should be folded and taped securely to prevent scratches. Teacher supervision is most important during mesh cage construction, in which the mesh will likely be cut again, exposing frayed edges. Additionally, the hot glue gun might pose the risk of minor burns.

## *Getting Ready*

Lay out the different types of meshes, such that students can easily decide which types of mesh to incorporate in their Faraday cages.

## *Introducing the Activity*

Review the concepts of static electricity, electric fields, and conductors, referring to the **introductory reading** included with this packet. This reading can offer an understanding of Faraday cages and electromagnetic shielding.

* For students without prior background in specific science topics, assign students to read Parts I and/or II of the **introductory reading**, which cover these topics in sufficient depth to understand this exploration.

## *Procedure*

1. After explaining the appropriate background concepts, introduce the activity by telling students that they can make a radio turn on or off without actually touching it.
2. Give each group of students a cardboard box.
3. Explain to the students how different thicknesses and mesh sizes will affect the cage. (See **introductory reading**, Part II).
4. Let the students choose how many layers to incorporate into their Faraday cage, and ask them to predict whether the cage will cancel FM radio signals, AM radio signals, or both types.
5. Have the students cover the cardboard box with the mesh. Ensure that students tape the mesh to every side of the box, making sure that all areas are covered such that when closed, the metal surface of the box is contiguous. The result is a Faraday enclosure.

## *Time-Saving Procedure*

1. Instead of having each group of students build their own Faraday cages, prepare the cages beforehand and allow the students to choose which cage(s) they would like to test out.
2. Prepare different cages with varying numbers of mesh layers, as well as cages with large holes in the mesh walls. If necessary, label the cages so that the students can clearly differentiate between cages of different layers/continuity.
3. Then let the students test out the different cages and examine trends in the results.

### TESTING THE CAGE: DEMONSTRATION EXAMPLE

If appropriate, perform the following steps on a pre-built Faraday cage to give students an example of how to test their own cages. Otherwise, ask students to perform the following steps on their own, writing down and/or discussing answers to leading questions within their three-student groups.

1. First, ask the students what they think will happen when a radio is placed inside the cage.
2. Outside of the cage, turn the radio on and set it to an FM radio station that can be clearly heard.
3. Place the radio inside the box, and close the lid of the box. What happens, and why?
4. Now, while the radio is still inside the cage, extend its antenna to touch the inner mesh cage. Note any difference in performance, and ask students to explain what they think might be causing it.
5. Next, take the radio outside of the box and tune the radio to an AM station.
6. Place the radio back in the box and put the lid back on. Does the Faraday cage still block out these radio signals? Why or why not?
7. Lastly, test a cell phone in the box. Call a cell phone with another phone. Turn on the cell phone’s speaker function, and have one student talk through the other phone.
8. Place the cell phone inside the box and see if the student’s voice is still heard on the speakerphone.
9. Repeat as many of steps 1-9 as time permits, using Faraday cages with both single and multiple mesh layers, and even cages with large holes cut out of the mesh walls.
10. How effective was the Faraday cage in each scenario? Where there discrepancies between cages? Why or why not?

## *Explanation*

After students have tested their cages and begun to develop explanations about cage performance, use the following questions to ensure a proper understanding of the concepts explored in the hands-on activity. Ask the students to discuss the answers to the questions first in small groups and then with the entire class to make sure students can accurately answer the questions.

1. What is a conductor? What makes a good conductor?
2. How do electrons move in a conductor in response to an electric field?
3. How do electrons move in a conductor in response to an electromagnetic wave? (Consider the cell phone and radio examples.)
4. What is a Faraday Cage, and how does it act as a “shield”?

## *Optional Exercises*

**Explaining How Charges Move in Relation to Each Other:**
To explain the idea of charges moving away from each other in a conductor, clear the classroom and tell all the students to stand in the center of the room. Then, instruct them to move around as quickly as they can without bumping into their classmates. After a few minutes of movement, ask the class to observe how the students tend to move away from each other; most should move from the center of the room to the edge. At the end, explain a comparison between how the students moved away from each other in the room to electron repulsion in a conductor. Just like the students moved away from each other to the edge of the room, electrons also move away from each other in response to an electric field or incoming electromagnetic wave.

**Explaining How Opposite Charges Attract:**
To explain how opposite charges attract each other, rub a balloon on a student's head. Describe how the balloon is depositing negatively-charged electrons on the student's head. This makes the student's head negatively charged while the balloon is positively charged. Then lift the balloon from the student's head. The hair is attracted to the balloon and thus rises toward it.

**Explaining How Similar Charges Oppose:**
To explain how similar charges oppose each other, use a Van der Graaf generator. Turn the Van der Graaf on and have a student stand on a glass platform, rubber sheet, or any other insulator. (The surface should be an insulator so that the student does not "lose the electrons" to the floor). Then, ask the student to touch the Van der Graaf. The student's hair should stand on end because the metal part of the Van de Graff is always generating negative charge, and by touching the Van de Graff, the student becomes negatively charged and causes the student's hair to "run away" from the other negative charge.

**Explaining Wave Motion:**
To explain how waves propagate through a medium, have students shake a rope or a Slinky as a means of visualizing the motion of an electromagnetic wave. If teachers have access to a wave machine, consult **Appendix B** for further instruction.

## *Cross-Curricular Integration*

**Math:**
Report the dimensions of each box. Then ask the students to calculate how much mesh they would need to completely cover the box.
If a sheet of 1' x 1' mesh cost $1.75, how much would all the mesh cost? If you paid for the mesh with a $100 dollar bill, how much money should you get back?

**English:**
Ask the students to imagine and then write a story about being trapped in a Faraday cage. What would happen to any electronic devices they were holding? Maybe they would be trapped in an elevator or a metal parking structure. How would they get out without any way to contact the outside world? How did they get trapped inside in the first place?

***— End of instructor copy —***

### *Obstacles Encountered in Preceding Activity Development*

Naturally, the development of the preceding activity underwent several revisions. To begin, our original lesson plan included a large demonstration in which a student enclosed in a Faraday cage would remain unharmed when subjected to an electric shock. However, this demonstration would be costly and, more importantly, potentially dangerous if the experimental set-up was not carried out properly. Ultimately, we discarded this demonstration in light of the scope of our project. Next, when actually testing the Faraday cage by placing a radio inside it, we found that several seemingly random radio stations weren't blocked. Yet, upon reviewing the principles that govern Faraday cage effectiveness, we found that the strength of the signal can alter the degree to which signals are (or are not) blocked. This helped us clarify our explanations for the Faraday cage results in our activity.

Another concept we had to account for was the difference in FM and AM radio signal shielding. After conducting extensive background research, we found that high-frequency waves are more susceptible to conductor surface reflection (see Appendix C). Moreover, FM radio signals consist of stationary charges that are blocked by the Faraday cage. However, electric fields that are produced by the AM radio signals are time varying and thus do not get blocked by the Faraday cage [6]. After evaluating the depth of physics content that our activity introduces, we decided that this concept was too complex to explore in our hands-on activity and supplementary materials.

### *Time Valuation of the Hands-on Activity*

Some physics educators claim that "motivating strategies" comprise the most important component of an effective lesson. Examples of these strategies include "allowing students to make choices or autonomous decisions…create finished products, and…interact with peers" [7]. Although allowing students to build their own Faraday cages in the preceding activity takes more time, without expanding upon the pure physics concepts conveyed in a teacher-directed demonstration, the hands-on activity allows students to be more invested in the project. Educators profess that successful projects give students this type of "ownership of [a] problem or task," so that they feel personally involved in the activity and have a greater motivation to draw meaning from the learning process [8]. Accordingly, our Faraday cage activity allows students to build and test their own cages in small groups, fostering important interpersonal skills such as communication, teamwork, and design.

## *Project Reflection*

After completing the Olin Teach-In operation, our group has gained a deeper understanding of several subject areas within electromagnetic physics, as well as learned several important pedagogical lessons. Upon comparing different physics teaching methods, we found that our specific subject matter lent itself to teaching by analogy. We hope that our combination of analogy-based explanations, visual models, and hands-on exploration will serve as an example of effective teaching practices for abstract concepts such as electricity and magnetism.

## Appendix A:

## Activity Hand-out

Name: __________________
Partner(s) Name: __________________
__________________

# Faraday Cage Activity Sheet

Describe your cage. What material is your cage made out of? How many layers of your material do you have? Are there mesh holes or not? How big are the holes in your mesh?

Mark whether or not the following waves penetrate your cage.

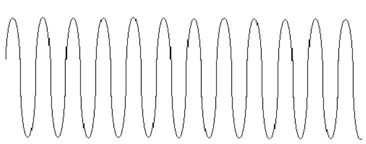
FM radio waves: _____

AM radio waves: _____

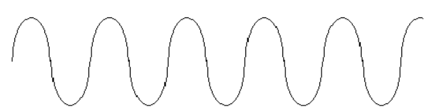
Baby monitor: _______

Cell Phone: __________

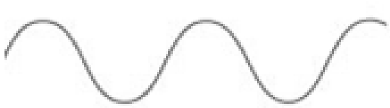
Cell phone frequency: 800 MHz



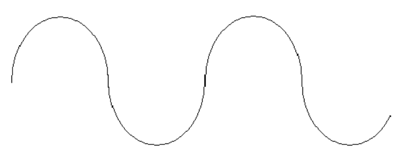
FM radio frequency: 88-108 MHz



Baby monitor: 49MHz



AM radio frequency: 535-1605 kHz

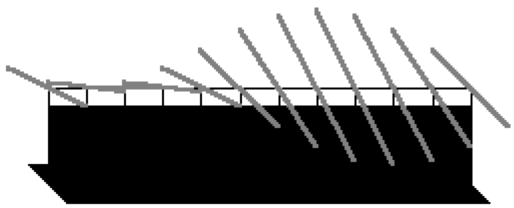


## Appendix B:

## Companion Wave Demonstration

### *If teachers have access to a wave machine*

A wave machine consists of numerous metal rods that lie in a row, horizontal to each other (shown below). All the rods are connected to each other so the movement of one rod will cause the movement of the rod next to it, resulting in wave propagation. Thus, this type of machine can be used to illustrate different properties of waves.



The most basic wave to demonstrate with this machine is a transverse wave, which travels in a direction perpendicular to the direction of the oscillations that produce it. To show this on the wave machine, ask a student volunteer to move the first metal rod up and down. Waves will propagate in the horizontal direction. Then, draw attention to the fact that each rod only moves in the vertical direction.

The next property to illustrate is wave reflection. To explain this phenomenon, ask one student to hold the last rod of the wave machine firmly in place. Then, when the first rod is moved up and down and the wave travels down the line of rods, the wave will hit the end and then get reflected back to its original starting place, as shown below. This defines wave reflection. The reason this happens is because when the wave gets to the fixed end on the right, it exerts an upward force. However, because this end is fixed, by Newton's third law, a downward force is generated. The wave and its reflected wave must have opposite signs so that their forces will cancel at the fixed end [9].

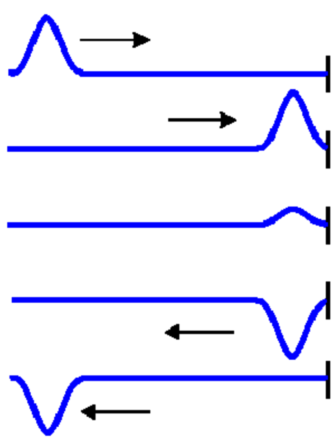


**Figure 1: A wave travels towards the wall on the right hand side of the diagram. When it reaches the fixed end point, it gets reflected in a direction opposite to that which it began. Therefore because the wave began in the upward direction, it ends up in the downward direction [10].**

Lastly, it is useful to demonstrate what happens when two waves meet. This can be shown by explaining how waves can cancel each other, which will happen when two waves meet each other and their amplitudes are equal in magnitude, but opposite in sign. For example, if two waves are equal in amplitude, but one first peaks in the upward direction and the other peaks in the downward direction, these two will cancel each other out. This is shown in Figure 2.

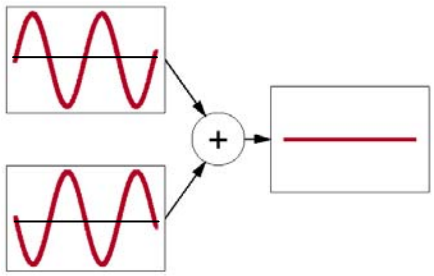



**Figure 2: The figure above illustrates how waves can cancel each other out. Notice how the waves are equal in amplitude, but opposite in direction. When these two waves meet, they form a straight line due to cancellation [11].**

This cancellation ensues because when two waves meet, the amplitude of the resultant wave is the sum of both component amplitudes. Therefore, if the two waves are equal in magnitude, but opposite in sign, they will add up to zero and cancel each other out [9]. In order to show this on the wave machine, it is crucial that two waves students send from either end of the machine have roughly equal but opposite amplitudes. The best way to do this is by trial and error. Ask the students to produce waves at different time intervals, and observe whether or not the waves cancel each other out when they meet in the center of the machine.

# Appendix C:

# Quantitative Explanations of Faraday Cage Shielding Phenomena

The objective of writing this appendix was to attain a quantitative understanding of electromagnetic waves and their behavior at a conducting surface. This resource may also be useful for instructors who wish to see the mathematical basis of our qualitative descriptions in the **introductory reading**.

## *Introduction to Maxwell's Equations [12-14]*

Maxwell's equations describe the behavior of electric and magnetic fields in relation to their sources, charge density ($\rho$), and current density (**J**). Comprised of four laws, the set of equations can be used to describe all phenomena in classical electromagnetism. Maxwell's equations are shown below, where **E** represents the electric field and **B** represents the magnetic field.

Maxwell's Equations:

| | |
|---|---|
| Gauss's law for **E**: | $\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$ |
| Gauss's law for **B**: | $\nabla \cdot \boldsymbol{B} = 0$ |
| Faraday's Law: | $\nabla \times \mathbf{E} = -\frac{d\phi_B}{dt}$ |
| Ampere's Law: | $\nabla \times \mathbf{B} = \mu_0(\boldsymbol{J} + \epsilon_0 \frac{d\phi_E}{dt})$ |

Maxwell's Equations in a Vacuum:

| | |
|---|---|
| Gauss's law for **E**: | $\nabla \cdot \boldsymbol{E} = 0$ |
| Gauss's law for **B**: | $\nabla \cdot \boldsymbol{B} = 0$ |
| Faraday's Law: | $\nabla \times \mathbf{E} = -\frac{d\phi_B}{dt}$ |
| Ampere's Law: | $\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$ |

Gauss's law tells us that the divergence (the dot product of $\nabla$ and some vector) of an electric field is proportional to the charge density at the given point. Divergence describes the tendency of the field to act as a source or a sink. While electric fields begin and end at their source, an electric charge, magnetic fields circle their sources. Consequently, the divergence of the magnetic field must be zero. Magnetic monopoles do not exist, as a magnetic north or south pole cannot be isolated. Thus, the net number of magnetic field lines through some closed surface is always zero.

Faraday's law shows us that the source of the magnetic field is moving electric charge (changing electric fields). Curl (the cross product of $\nabla$ and some vector) describes the rotation of a field. Thus, the curl gives us some measure of how the electric field is changing. This is shown to be equal to the negative rate of change of magnetic flux. Keep in mind that flux is a measure of quantity of the magnetic field – we can think of it in terms of the number of field lines through the closed surface.

Finally, Ampere's law completes the symmetry of Maxwell's equations. Ampere's Law, in its original form is as follows:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I$$

However, this does not reflect happens when there is a changing current present in the system. In such an event, the current through any open surface is not equal over all the surfaces, resulting in unresolved ambiguity. Maxwell further developed this formula, suggesting that a changing electric flux should produce a magnetic field. This gave rise to the differential form of Ampere's Law shown at the introduction of this section, which *does* account for non-steady currents.

Thus, the additional $\epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$ term is known as the *displacement current* ($\boldsymbol{J}_d$). Though not an actual current, its existence gives Maxwell's equations its symmetry – just as a changing magnetic field induces an electric field, a changing electric field induces a magnetic field.

## *Electromagnetic Waves* [12-15]

A wave may be defined as "a disturbance of a continuous medium that propagates with a fixed shape at constant velocity." Recall that Faraday's law demonstrates that a changing magnetic field induces an electric field, while Ampere's law shows that a changing electric field induces a magnetic field. Taken together, these two laws show that the changing fields continually produce another, resulting in some disturbance in space. This qualitatively suggests the existence of electromagnetic waves – how do we prove that they really exist?

Let us discuss plane electromagnetic waves. For simplicity, let us examine their propagation in a vacuum. The relevant Maxwell's equations are as follows:

$$\begin{aligned} \nabla \cdot \boldsymbol{E} &= 0 \\ \nabla \cdot \boldsymbol{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{d\phi_B}{dt} \\ \nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{d\phi_E}{dt} \end{aligned}$$

By applying the curl to the differential forms of Faraday's law and Ampere's law, we find the following, two identical equations consistent with the standard wave equation:

$$\nabla^2 \boldsymbol{E} = \mu_0 \epsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} \qquad \text{and} \qquad \nabla^2 \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial^2 \boldsymbol{B}}{\partial t^2}$$

Solutions to the differential equation for the electric field can be written as a plane wave, varying in one spatial direction with time. We can construct the electric field:

$$\boldsymbol{E} = \boldsymbol{E_0} f(kz \pm wt)\hat{\boldsymbol{x}} \,,$$

where **E** is parallel to the x-axis with an intensity (measured by $\mathbf{E_0}$) dependent on space-coordinate *z*. We can assign a corresponding magnetic field with only a y-component:

$$\boldsymbol{B} = \boldsymbol{B_0} f(kz \pm wt)\hat{\boldsymbol{y}}.$$

In this case, $k$ is the wave number, which describes the number of wave cycles per unit length $(1/\lambda)$, and $w$ is angular frequency.

We can show that these equations hold for Maxwell's equations. The curl of the electric field results in the following:

$$\frac{\partial E_x}{\partial z}\hat{\jmath} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

In taking the appropriate partial derivatives, we get $kE_0\hat{\jmath} = w\boldsymbol{B}_0$. The ratio of the amplitude of the electric field to that the magnetic field is equal to $\frac{w}{k}$, so we find that the Faraday's law is, in fact, satisfied. The same manipulations can be done to show that Ampere's law holds true as well.

Thus, we have solutions to the wave equations that are consistent with Maxwell's equations, proving the existence of electromagnetic waves.

Because the decoupled equations for the magnetic and electric fields are equal, it stands to reason that the two resulting waveforms are in phase since their relative positions have the same time-dependence. The conditions governing our equations were such that the electric and magnetic fields are perpendicular to each other and the direction of travel. The fields are not dependent on the direction of propagation, so it follows that the perpendicularity is the case. The two waveforms formed by the changing electric and magnetic field are in phase – thus, the proportionality between the field strengths is equal at any point in time.

Now that we have a basic understanding of electromagnetic waves, we can discuss how they are affected by conducting surfaces and other boundary conditions.

## *Electromagnetic Shielding*

We qualitatively understand that an electromagnetic wave approaching a conducting surface changes the distribution of charges such that accelerating electrons on the conducting surface induce another electromagnetic disturbance. How, then, do we prove that the "reflected wave" does cancels the incident wave? The following calculations will illustrate that a conductor limits how far a plane wave can penetrate the conducting surface. Then, we will describe how an incident plane wave is reflected off the surface of a closed conductor, effectively making the conductor an electromagnetic shield.

## *Quantitative Distinctions between Effective and Ineffective Conductors* [13]

According to Ohm's Law, the current density $\boldsymbol{J}$ in a conductor is proportional to the conductor's external electric field:

$$\boldsymbol{J} = \sigma\mathbf{E}$$

where $\sigma$ describes the conductivity of the conductor—a measure of how well a material can conduct electricity under certain conditions.

Current density is also incorporated in the *continuity equation for free charge*, an equation that expressed charge conservation—the principle that electric charge cannot be created or destroyed:

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$$

where $\rho$ is free charge density in the conductor. Now, we can combine the continuity equation with Gauss's law ($\nabla \cdot \boldsymbol{E} = \rho/\epsilon$ in a homogenous medium) to obtain an expression for free charge density dissipation in a conductor:

$$\nabla \cdot \boldsymbol{J} = \sigma\,(\nabla \cdot \boldsymbol{E})$$
$$-\frac{\partial \rho}{\partial t} = -\sigma(\nabla \cdot \boldsymbol{E}) = -\frac{\sigma}{\epsilon}\rho$$

An analytical solution for the differential equation $-\frac{\partial \rho}{\partial t} = -\frac{\sigma}{\epsilon}\rho$ is given by:

$$\rho(t) = e^{-\left(\frac{\sigma}{\epsilon}\right)t}\rho(0)$$

This equation illustrates that any initial free charge density $\rho(0)$ present in a conductor dissipates to the edges of the conducting surface within a time period $\tau = \epsilon/\sigma$ , after which $\rho$ returns to zero. (This also cancels the right hand form of Gauss's law in a conductor: $\nabla \cdot \boldsymbol{E} = 0$.)

As the conductivity $\sigma$ of the conductor approaches infinity (corresponding to an increasingly effective conductor), this time constant $\tau$ approaches zero, illustrating that electricity moves instantaneously in perfect conductors. However, in actual systems such as our Faraday cages, $\tau$ must only $\ll 1/\omega$ (where $\omega$ is the angular frequency of an incoming wave), for the cage to be able to respond to the incoming electromagnetic wave.

### *Skin Depth and Wave Penetration into a Conductor* [13,16,17]

When we apply the curl to the $\boldsymbol{E}$ and $\boldsymbol{B}$ wave equations, we arrive at the following modified equations:

$$\nabla^2 \boldsymbol{E} = \mu\epsilon\frac{\partial^2 \boldsymbol{E}}{\partial t^2} + \mu\sigma\frac{\partial \boldsymbol{E}}{\partial t} \quad \text{and} \quad \nabla^2 \boldsymbol{B} = \mu\epsilon\frac{\partial^2 \boldsymbol{B}}{\partial t^2} + \mu\sigma\frac{\partial \boldsymbol{B}}{\partial t}$$

The above equations present the following plane-wave solutions:

$$\boldsymbol{E}(z,t) = \boldsymbol{E_0}e^{i(kz-\omega t)} \quad \text{and} \quad \boldsymbol{B}(z,t) = \boldsymbol{B_0}e^{i(kz-\omega t)}$$

Note that the wave number $k$ is complex in these solutions (indicated by the imaginary $i$ multiplier). If we substitute the equation for $\boldsymbol{E}(z,t)$ into the above equation for $\nabla^2\boldsymbol{E}$, we obtain the following expression:

$$k = k + i\kappa$$

where $\kappa$ in the imaginary component of $k$ is the attenuation constant of the wave, which causes the wave to decrease in amplitude as the wave continues to propagate—further along the positive $z$ axis, with the given plane waves. This wave attenuation is incorporated into the plane equations as follows:

$$\boldsymbol{E}(z,t) = \boldsymbol{E_0}e^{-\kappa z}e^{i(kz-\omega t)} \quad \text{and} \quad \boldsymbol{B}(z,t) = \boldsymbol{B_0}e^{-\kappa z}e^{i(kz-\omega t)}$$

From these equations, we can characterize a unit of length $d$ for a given plane wave, over which the wave has to travel for its amplitude to decrease by a factor of $1/e$:

$$d = \frac{1}{\kappa}$$

The value of $d$ is termed **skin depth**, a property of conductive material that measures how far a plane wave can penetrate into the conductor surface. For a given conductor, $\kappa$ increases with higher incident wave frequency. This translates to a lower skin depth $d$, so a high frequency wave can only propagate through a relatively small conductive region before it diminishes. In turn, high frequency waves encounter more electrical resistance because of the decreased surface area through which they can propagate.

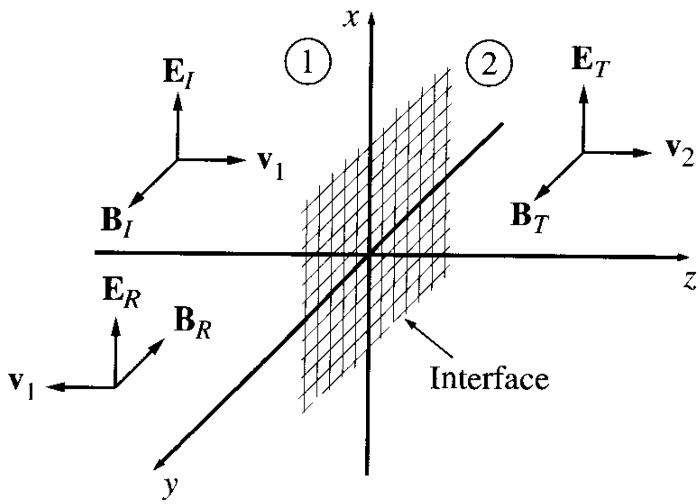
## *Wave Reflection at the Surface of a Conductor* [13]

Now, consider a configuration in which the $xy$ plane is a boundary between (1) an insulator (for example, air) and (2) a conductor (for example, a Faraday enclosure). An incident monochromatic plane wave, or a wave with only one frequency component, propagates in the positive $z$ direction, and is polarized in the $x$ direction:

$$\boldsymbol{E_I}(z,t) = E_{0I}e^{i(k_1 z-\omega t)}\hat{\boldsymbol{x}} \quad \text{and} \quad \boldsymbol{B_I}(z,t) = \frac{1}{v_1}E_{0I}e^{i(k_1 z-\omega t)}\hat{\boldsymbol{y}}$$

In Figure 1, this incident wave is indicated by the terms subscripted with an $I$, while terms with a $R$ subscript represent the reflected wave, and the $T$ subscript represents the final wave transmitted into the conductive medium.

**Figure 1**




The given incident wave results in a reflected wave:

—

This reflected wave propagates in the opposite direction (back into the first nonconducting medium), indicated by the – terms. The incident wave also produces a transmitted wave:

—

This time, the positive term indicates that the transmitted wave propagates in the same direction as the incident wave, as shown in Figure 1. This transmitted wave is attenuated while it penetrates the conductor.

When , both waves in medium 1 join the wave in medium 2. The following equations define our boundary conditions at the interface:

$$\text{(i)}\ \epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f, \quad \text{(iii)}\ \mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = 0,$$

$$\text{(ii)}\ B_1^{\perp} - B_2^{\perp} = 0, \quad \text{(iv)}\ \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} - \frac{1}{\mu_2}\mathbf{B}_2^{\parallel} = 0$$

Because no or in our system are perpendicular to the interface, boundary condition i is zero, and condition ii is satisfied. Then, condition iii yields the following relationship between the parallel to the interface in each medium:

Finally, substituting $\boldsymbol{B_I}$ and $\boldsymbol{B_R}$ for $B_1$ and $\boldsymbol{B_T}$ for $B_2$ in condition iv yields the following relationship:

$$\frac{1}{\mu_1 v_1} = (E_{0I} - E_{0R}) - \frac{k_2}{\mu_2 \omega} E_{0T} = 0$$

If we define $\beta$ as follows:

$$\beta = \frac{\mu_1 v_1}{\mu_2 \omega} k_2 \,,$$

then,

$$E_{0I} - E_{0R} = \beta E_{0T}$$

We can also substitute $\beta$ into the above expressions for $\boldsymbol{E}$:

$$E_{0R} = \left(\frac{1-\beta}{1+\beta}\right) E_{0I} \quad \text{and} \quad E_{0T} = \left(\frac{2}{1+\beta}\right) E_{0I}$$

For a perfect conductor, where the conductivity $\sigma$ is infinite, $k_2$ is also infinite, meaning that $\beta$ is also infinite. Thus, taking $\beta$ to be infinite:

$$E_{0R} = -E_{0I} \quad \text{and} \quad E_{0T} = 0$$

These relations correspond to a perfect 180º phase shift between the incident and reflected waves. Thus, no plane wave is transmitted into medium 2 (the conductor), making the conductor a “shield” to the electromagnetic wave. Although our Faraday cage is not a perfect conductor, its conductivity provides near reflection of an incident plane wave (for instance, a radio signal), thereby cancelling incoming signals.

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